Name

Functions and their Inverses | 1.5

Ready, Set, Go!

Ready

Topic: Properties of exponents

Use properties of exponents to simplify the following. Write your answers in exponential form with positive exponents.

1. \( \sqrt[3]{x^2} \cdot \sqrt[3]{x^3} = \sqrt[3]{x^5} \)

2. \( \sqrt[3]{x} \cdot \sqrt[3]{x} \cdot \sqrt[5]{x} = x^{\frac{1}{3} + \frac{1}{4} + \frac{1}{6}} \)

3. \( \sqrt[6]{a} \cdot \sqrt[6]{a^2} \cdot \sqrt[6]{b^3} \)

4. \( \sqrt[3]{32} \cdot \sqrt[3]{9} \cdot \sqrt[3]{27} = 2 \cdot 3 \cdot 3 \)

5. \( \sqrt[6]{8} \cdot \sqrt[16]{16} \cdot \sqrt[2]{2} \)

6. \( (5^2)^3 \)

7. \( (7^2)^{-1} \)

8. \( (3^{-4})^{-5} \)

9. \( \left( \frac{5^2}{3^2} \right)^3 \)

Set

Topic: Representations of inverse functions

Write the inverse of the given function in the same format as the given function.

<table>
<thead>
<tr>
<th>Function ( f(x) )</th>
<th>Inverse ( f^{-1}(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.</td>
<td></td>
</tr>
<tr>
<td>( x )</td>
<td>( x )</td>
</tr>
<tr>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
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<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
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</tbody>
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<th>( f^{-1}(x) )</th>
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<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
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<td>6</td>
<td>0</td>
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<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>
12. \( f(x) = -2x + 4 \)

\[ f^{-1}(x) = \frac{-x + 4}{2} \]

13. \( f(x) = \log_3 x \)

\[ f^{-1}(x) = 3^x \]

15. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>4</td>
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<td>16</td>
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</tr>
</tbody>
</table>
Calculate \( f(g(x)) \) and \( g(f(x)) \) for each pair of functions.
(Note: the notation \( f \circ g)(x) \) and \( (g \circ f)(x) \) mean the same thing, respectively.)

16. \( f(x) = 3x + 7; \ g(x) = -4x - 11 \)
\[
\begin{align*}
f(g(x)) &= 3(-4x - 11) + 7 \\
&= -12x - 33 + 7 \\
&= -12x - 26 \\
g(f(x)) &= -4(3x + 7) - 11 \\
&= -12x - 28 - 11 \\
&= -12x - 39
\end{align*}
\]

17. \( f(x) = -4x + 60; \ g(x) = -\frac{1}{4}x + 15 \)
\[
\begin{align*}
f(g(x)) &= -4(-\frac{1}{4}x + 15) + 60 \\
&= x - 60 + 60 \\
&= x \\
g(f(x)) &= -\frac{1}{4}(-4x + 60) + 15 \\
&= x - 15 + 15 \\
&= x
\end{align*}
\]

18. \( f(x) = 10x - 5; \ g(x) = \frac{2}{5}x + 3 \)
\[
\begin{align*}
f(g(x)) &= 10(\frac{2}{5}x + 3) - 5 \\
&= 4x + 30 - 5 \\
&= \frac{4}{5}x + 25 \\
g(f(x)) &= \frac{2}{5}(10x - 5) + 3 \\
&= 4x - 2 + 3 \\
&= 4x + 1
\end{align*}
\]

19. \( f(x) = -\frac{2}{3}x + 4; \ g(x) = -\frac{3}{2}x + 6 \)
\[
\begin{align*}
f(g(x)) &= -\frac{2}{3}\left(-\frac{3}{2}x + 6\right) + 4 \\
&= x - 4 + 4 \\
&= x \\
g(f(x)) &= -\frac{3}{2}\left(-\frac{2}{3}x + 4\right) + 6 \\
&= x - 6 + 6 \\
&= x
\end{align*}
\]

20. Look back at your calculations for \( f(g(x)) \) and \( g(f(x)) \). Two of the pairs of equations are inverses of each other. Which ones do you think they are?

Why?

17 and 19  b/c  \( f(g(x)) = g(f(x)) \)